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WAVE FORMATION IN LIQUID FILM FLOW ON A VERTICAL WALL

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Theoretical and experimental investigations show that laminar vertical liquid film flow is unstable starting with the lowest Reynolds numbers Re. The instability results in the origination of periodic waves, which grow rapidly in amplitude with distance and emerge into the stationary mode for specific amplitudes. Linear stability of a smooth film was investigated in many papers [1-5]. The greatest successes have been achieved on the basis of numerical methods of calculating the Orr–Sommerfeld equation. Dependences have been obtained for the wave amplitude increment, for the phase velocity and wave number of neutral perturbations and maximum growth waves as a result of the computations.

Clarity in the nonlinear wave formation mechanism at high Reynolds numbers is substantially lower. Research on nonlinear waves can be divided into two provisional groups in which the cases of low and high numbers Re are examined, respectively. For the case Re ~ 1 (here $Re = q_0/\nu$, q_0 is the specific mass flow rate, and ν is the kinematic viscosity), a nonlinear nonstationary equation is derived for long waves on the film surface by using the method of narrow bands [6-8]. For the moderate number range Re ~ 5-50, only a stationary equation is derived [9-11], and nonlinear nonstationary waves are analyzed on the basis of a system of equations of boundary layer type by using the integral relations method.

There is a definite objective need to derive a universal model equation for nonstationary nonlinear waves which would permit extension of existing approaches and which would be valid in a broad range of numbers Re. An attempt to derive such an equation is presented in this paper. Results of a linear analysis of this equation are compared with experimental results for growing linear waves and with results of other authors.

1. DERIVATION OF THE EQUATION FOR THE WAVES

Let us write the Navier–Stokes equations and the boundary conditions for a fluid film flowing on a vertical wall (Fig. 1) in the dimensionless form

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{3}{\mathrm{Re}\,\varepsilon} + \frac{1}{\varepsilon\,\mathrm{Re}}\left(\frac{\partial^2 u^*}{\partial x^{*2}}\varepsilon^2 + \frac{\partial^2 u^*}{\partial y^{*2}}\right) - \frac{\partial p^*}{\partial x^*}; \tag{1.1}$$

$$\varepsilon^2\left(\frac{\partial v^*}{\partial t^*} + u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \frac{3}{\mathrm{Re}}\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \varepsilon^2\frac{\partial^2 v^*}{\partial x^{*2}}\right) - \frac{\partial p^*}{\partial y^*}; \tag{1.2}$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0; \tag{1.3}$$

$$\varepsilon^2\frac{4\partial h^*/\partial x^*}{1-\varepsilon^2(\partial h^*/\partial x^*)^2}\frac{\partial v^*}{\partial y^*} + \frac{\partial u^*}{\partial y^*} + \varepsilon^2\frac{\partial v^*}{\partial x^*} = 0 \quad \text{for } y = h; \tag{1.4}$$

$$\Delta p^* = -\frac{3^{1/3}\mathrm{Fi}^{1/3}\varepsilon^2}{\mathrm{Re}^{5/3}}\frac{\partial^2 h^*/\partial x^{*2}}{[1+\varepsilon^2(\partial h^*/\partial x^*)^2]^{3/2}} + \frac{2\varepsilon}{\mathrm{Re}}\frac{\partial v^*}{\partial y^*}\left[\frac{1+\varepsilon^2(\partial h^*/\partial x^*)^2}{1-\varepsilon^2(\partial h^*/\partial x^*)^2}\right]$$

for y = h;

$$u^* = 0, \quad v^* = 0 \quad \text{for } y = 0; \tag{1.6}$$

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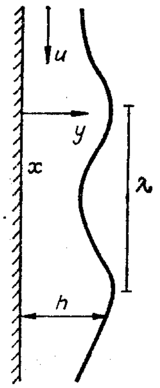

Fig. 1

$$v^* = \frac{\partial h^*}{\partial t^*} + U^* \frac{\partial h^*}{\partial x^*} \quad \text{for} \quad y = h, \tag{1.7}$$

where U* is the dimensionless longitudinal velocity component on the film surface. Conditions (1.4) and (1.5) indicate the absence of tangential and normal stresses on the film surface, while condition (1.7) is the usual kinematic condition on a free surface. The following dimensionless quantities have been introduced here

$$u^* = u/u_0,\ v^* = (v/u_0)L/h_0,\ x^* = x/L,\ y^* = y/h_0,$$
$$t^* = tu_0/L, \quad p^* = p/(\rho u_0^2), \quad \varepsilon = h_0/L,$$

$Fi = \sigma^3/(\rho^3 g \nu^4)$ is the film number, where t is the time, L is the characteristic longitudinal dimension (wavelength, for instance), p is the pressure, g is the free-fall acceleration, ρ is the density, σ is the fluid surface tension, and h_0 and u_0 are determined from the Nusselt formulas for smooth laminar film flow $Re = q_0/\nu = gh_0^3/(3\nu^2) = h_0 u_0/\nu$.

Let us examine the longwave process $\varepsilon \ll 1$ and setting $Re \sim 1/\varepsilon$ we keep terms of order 1 in (1.1)-(1.7). We consequently arrive at equations of the boundary layer type, which we write in dimensional form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} + g; \tag{1.8}$$

$$\frac{\partial p}{\partial y} = 0; \tag{1.9}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1.10}$$

with the boundary conditions

$$\frac{\partial u}{\partial y} = 0 \quad \text{for} \quad y = h; \tag{1.11}$$

$$\frac{\partial p}{\partial x} = -\sigma\frac{\partial^3 h}{\partial x^3} \quad \text{for} \quad y = h. \tag{1.12}$$

Conditions (1.6) and (1.7) remain unchanged. Using (1.9) and (1.12), we rewrite (1.8) in the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g + \frac{\sigma}{\rho}\frac{\partial^3 h}{\partial x^3}. \tag{1.13}$$

It was taken into account in the derivation of (1.13) that $Fi^{1/3}$ is a large quantity for real fluids (for instance, $Fi^{1/3} \approx 10^4$ for water).

It is important to note that the approximate equations (1.11) and (1.13) retain their form in a very much greater range of Re numbers than $Re \sim 1/\varepsilon$. For $Re \sim 1/\varepsilon$ all the terms of (1.13) are of the same order, for $Re \sim 1$ the inertial terms in (1.13) are less than all the remaining terms, while for $Re \sim 1/\varepsilon^2$ they are correspondingly greater. However, it is seen that the order of the discarded terms in the initial system is always less in the number range $Re = 1 - 1/\varepsilon^2$ than is the order of the terms retained in (1.13) and (1.11), i.e., the number Re can actually vary between one and numbers corresponding to the transition to the turbulent mode in the approximation mentioned.

We henceforth use the integral relations method (the Karman—Pohlhausen method). The main difficulty in this method is that the instantaneous velocity profile in the film must be given by an a priori method. It is difficult to estimate the error occurring here although it is intuitively clear that it should not be loo large in the long wave case. The results of experiments [12-13] on the direct determination of the instantaneous velocity profiles in a fluid wave film indicate a satisfactory approximation of the velocity profile by a self-similar polynomial in the case of two-dimensional waves of, at least, moderate amplitude.

By integrating (1.13) and (1.10) over the film thickness, we obtain after simple manipulations

$$\frac{\partial}{\partial t}\int_0^h u\,dy + \frac{\partial}{\partial x}\int_0^h u^2 dy = -\nu\left(\frac{\partial u}{\partial y}\right)_{y=0} + gh + \frac{\sigma h}{\rho}\frac{\partial^3 h}{\partial x^3}; \tag{1.14}$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\int_0^h u\,dy = 0. \tag{1.15}$$

Let us represent the velocity profile as

$$u = U f(\eta),\ \eta = y/h.$$

The function f can be approximated by a second degree polynomial, say, with coefficients satisfying the boundary conditions (1.6) and (1.11)

$$f(\eta) = 2\eta - \eta^2. \tag{1.16}$$

Furthermore, we introduce the instantaneous fluid mass flow rate in the film and express it in terms of f

$$q = \int_0^h u\,dy = Uh\int_0^1 f(\eta)\,d\eta.$$

Analogously, we have

$$\int_0^h u^2 dy = U^2 h\int_0^1 f^2(\eta)\,d\eta,\ \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{U}{h}\left.\frac{df}{d\eta}\right|_{\eta=0}.$$

Using these expressions and introducing the notation

$$\int_0^1 f\,d\eta = \delta,\ \int_0^1 f^2 d\eta = \gamma,\ \left.\frac{df}{d\eta}\right|_{\eta=0} = \varkappa,\quad \chi = \gamma/\delta^2,$$

we rewrite (1.14) and (1.15) in the form of equations for the thickness and the mass flow rate

$$\frac{\partial q}{\partial t} + 2\chi\frac{q}{h}\frac{\partial q}{\partial x} - \chi\frac{q^2}{h^2}\frac{\partial h}{\partial x} = -\frac{\varkappa\nu}{\delta h^2}q + gh + \frac{\sigma h}{\rho}\frac{\partial^3 h}{\partial x^3}; \tag{1.17}$$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0. \tag{1.18}$$

Let us represent the total flow in the form $q = q_0 + q'$, $h = h_0 + h'$, where the prime denotes the perturbed part of the quantity, and let us substitute these expressions in (1.17) and (1.18). Setting $q' \ll q_0$, $h' \ll h_0$, and keeping terms on the order of h'^2, q'^2 in the equations, we obtain nonlinear equations for the thickness and mass flow rate perturbations with nonlinear terms in the right side

$$\frac{\partial q'}{\partial t} + \frac{2\chi q_0}{h_0}\frac{\partial q'}{\partial x} - \chi\frac{q_0^2}{h_0^2}\frac{\partial h'}{\partial x} + \frac{\varkappa\nu}{\delta h_0^2}q' - 3gh' - \frac{\sigma h_0}{\rho}\frac{\partial^3 h'}{\partial x^3} = \frac{3gh'^2}{h_0} - \frac{2}{h_0}h'\frac{\partial q'}{\partial t} - \frac{2\chi}{h_0^2}q_0\left[h'\frac{\partial q'}{\partial x} + h_0\frac{q'}{q^0}\frac{\partial q'}{\partial x} - q'\frac{\partial h'}{\partial x}\right]; \tag{1.19}$$

$$\frac{\partial h'}{\partial t} + \frac{\partial q'}{\partial x} = 0. \tag{1.20}$$

Let us differentiate (1.19) with respect to x and let us replace the derivative $\partial q'/\partial x$ in the linear terms by means of the continuity equation (1.20), then

$$\frac{\partial^2 h'}{\partial t^2}+\frac{2\chi q_0}{h_0}\frac{\partial^2 h'}{\partial x\partial t}+\chi\frac{q_0^2}{h_0^2}\frac{\partial^2 h'}{\partial x^2}+\frac{\varkappa\nu}{\delta h_0^2}\frac{\partial h'}{\partial t}+3g\frac{\partial h'}{\partial x}+\frac{\sigma h_0}{\rho}\frac{\partial^4 h'}{\partial x^4}$$
$$=-\frac{6g}{h_0}h'\frac{\partial h'}{\partial x}+\frac{2}{h_0}\frac{\partial}{\partial x}\left(h'\frac{\partial q'}{\partial t}\right)+\frac{2\chi q_0}{h_0^2}\frac{\partial}{\partial x}\left[h'\frac{\partial q'}{\partial x}+\frac{h_0}{q_0}q'\frac{\partial q'}{\partial x}-q'\frac{\partial h'}{\partial x}\right]. \tag{1.21}$$

In order to eliminate q' and $\partial q'/\partial t$ from the nonlinear terms, we use the following considerations. We go from the variables x, t in the continuity equation to the variables ξ, t, where $\xi = x - ct$, c is the propagation velocity of the perturbations which we shall consider constant for the quasistationary wave case. As follows from experiment, in many cases the waves under consideration can actually be considered as weakly dispersive and weakly nonlinear. We then have

$$\partial h'/\partial t - c\partial h'/\partial \xi + \partial q'/\partial \xi = 0. \tag{1.22}$$

In the case of a quasistationary process, the wave profile in the moving coordinate system is deformed slightly whereupon we arrive at the approximate equation $c\partial h'/\partial\xi = \partial q'/\partial\xi$ from (1.22), from which the relationships

$$q' = ch'; \tag{1.23}$$
$$\partial/\partial t = -c\partial/\partial x \tag{1.24}$$

follow. For stationary waves (1.23)-(1.24) are exact.

Now, let us substitute (1.23) into the nonlinear terms in (1.21), which are always of higher order for $\mathrm{Re} = 1 - 1/\varepsilon^2$ than the main terms. We replace derivatives of the form $c\partial/\partial x$ appearing here in the nonlinear terms according to (1.24). We thus obtain a nonlinear nonstationary equation for the thickness perturbation

$$\left(\frac{\partial}{\partial t}+c_0\frac{\partial}{\partial x}\right)h'+\frac{\delta}{\varkappa}\frac{h_0^2}{\nu}\left(\frac{\partial}{\partial t}+c_1\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}+c_2\frac{\partial}{\partial x}\right)h'$$
$$+6\frac{\delta h_0 g}{\varkappa\nu}h'\frac{\partial h'}{\partial x}-\frac{2\delta}{\varkappa}(\chi-1)\frac{h_0}{\nu}\frac{\partial}{\partial t}\left(h'\frac{\partial h'}{\partial t}\right)+\frac{\delta\sigma}{\varkappa\rho\nu}h_0^3\frac{\partial^4 h'}{\partial x^4}=0, \tag{1.25}$$

where $c_0 = 3q_0/h_0$, $c_1 = q_0(\chi + \sqrt{\chi^2 - \chi})/h_0$, $c_2 = q_0(\chi - \sqrt{\chi^2 - \chi})/h_0$.

Equation (1.25) has the characteristic two-wave structure. This means that the wave process on the fluid film includes a kinematic wave, as a lower order wave with the velocity c_0, and waves described by higher order derivatives and being propagated with the velocities c_1 and c_2. The method of deriving and analyzing such equations is discussed in detail in [14].

For the subsequent discussions, let us use the approximation (1.16) for the velocity profile in a gravitationally flowing vertical fluid film, and let us evaluate the coefficients

$$\delta = 2/3,\quad \varkappa = 2,\quad \chi = 1.2,\quad c_1 = 1.69\,u_0,$$
$$c_2 = 0.71u_0,\quad u_0 = q_0/h_0 = gh_0^2/(3\nu).$$

Substituting these values into (1.25) and nondimensionalizing, we finally have

$$\left(\frac{\partial}{\partial t^*}+3\frac{\partial}{\partial x^*}\right)H+6H\frac{\partial H}{\partial x^*}-\frac{2}{15}\mathrm{Re}\left(\frac{h_0}{L}\right)\frac{\partial}{\partial t^*}\left(H\frac{\partial H}{\partial t^*}\right)+\frac{\mathrm{Re}}{3}\left(\frac{h_0}{L}\right)\left(\frac{\partial}{\partial t^*}\right.$$
$$\left.+1.69\frac{\partial}{\partial x^*}\right)\left(\frac{\partial}{\partial t^*}+0.71\frac{\partial}{\partial x^*}\right)H+\mathrm{We}\left(\frac{h_0}{L}\right)^3\frac{\partial^4 H}{\partial x^{*4}}=0, \tag{1.26}$$

where $H = h'/h_0$. It is always possible to arrive at the standard manner with one scale h_0, from the form of writing (1.26), by assuming $L = h_0$.

If it is assumed that the wave is stationary, $h' = h'(x - ct)$, then (1.26) will correspond to the equations in [10, 11].

Let us consider the case $h_0/L \ll 1$, Re ~ 1. We see that underlying the wave process is the kinematic wave $\partial H/\partial t^* + 3\partial H/\partial x^* = 0$. Following [14], we replace the derivative with respect to the time in the higher order wave by using the relationship $\partial/\partial t^* = -3\partial/\partial x^*$, we neglect the second term, and we go from (1.26) to the equation

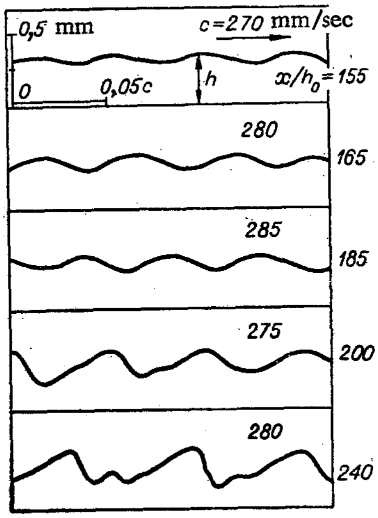

Fig. 2

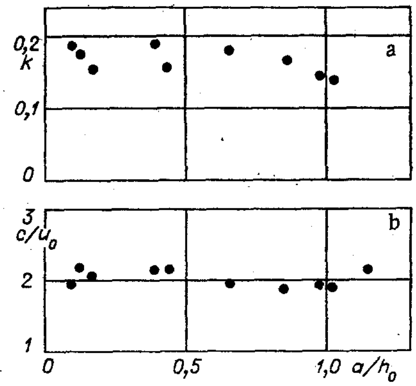

Fig. 3

$$\frac{\partial H}{\partial t^*}+3\frac{\partial H}{\partial x^*}+6H\frac{\partial H}{\partial x^*}+\mathrm{Re}\left(\frac{h_0}{L}\right)\frac{\partial^2 H}{\partial x^{*2}}+\mathrm{We}\left(\frac{h_0}{L}\right)^3\frac{\partial^4 H}{\partial x^{*4}}=0. \tag{1.27}$$

An equation of the type (1.27) was recently used in [7, 8, 15, 16] as the main equation in an analysis of nonlinear waves on a film.

Let us consider the other limit case when $\mathrm{Re}(h_0/L) \gg 1$. Underlying the wave process will be a second-order wave. Let us first extract the wave $\partial H/\partial t^* + 1.69\partial H/\partial x^* = 0$ being propagated along the stream. In all the remaining derivatives we substitute $\partial/\partial t^* = -1.69\partial/\partial x^*$ and we obtain

$$\frac{\partial H}{\partial t^*}+1.69\frac{\partial H}{\partial x^*}+2.07H\frac{\partial H}{\partial x^*}-\frac{4.01}{\mathrm{Re}}\left(\frac{L}{h_0}\right)H-\frac{9.2}{\mathrm{Re}}\left(\frac{L}{h_0}\right)H^2-3.06\frac{\mathrm{We}}{\mathrm{Re}}\left(\frac{h_0}{L}\right)^2\frac{\partial^3 H}{\partial x^{*3}}=0. \tag{1.28}$$

An equation of such form was obtained in [17] as a model equation for waves on a film surface for large Re.

Therefore, in the case of the longwave process at low Reynolds numbers, the energy from the meanflow is pumped into the kinematic wave by means of a high-order wave mechanism. This corresponds to the appearance of a pumping term with a second derivative in (1.27), or as is sometimes said, a term with "negative" viscosity. For high Reynolds numbers, the energy in a high-order wave, which can provisionally be called "inertial," is pumped by a kinematic wave, which corresponds to the appearance of a linear "low-frequency" pumping term in (1.28).

A conclusion on the exact domain of applicability of (1.26) and on the possibility of a passage to the limit in (1.26) to the case of high numbers Re can be made only on the basis of comparing the solutions of this equation with experiment and exact numerical solutions. Such a comparison is made below for linear waves.

2. STABILITY ANALYSIS OF FILM FLOW

Let us derive a dispersion equation for temporarily growing (damping) waves from (1.26). To this end, we represent H in the form $H = A_0 \exp[i(kx^* - \Omega t^*)\varepsilon^{-1}] = A_0 \exp[ik\varepsilon^{-1}(x^* - c^*t^*)] \exp \beta t^*\varepsilon^{-1}$, where $k = 2\pi h_0/\lambda$ is the real wave number, $\Omega = \omega + i\beta$ is the complex frequency made dimensionless in terms of h_0 and u_0, $c^* = c/u_0$ is the real part of the phase velocity. We substitute H in the linearized equation (1.26) and separating real and imaginary parts, we obtain

$$-c^*+3-\frac{2}{3}c^*\beta\,\mathrm{Re}+0.8\,\mathrm{Re}\,\beta=0; \tag{2.1}$$

$$3\beta-k^2\,\mathrm{Re}\,(c^{*2}-2.4c^*+1.2)+\beta^2\,\mathrm{Re}+3\,\mathrm{We}\,k^4=0. \tag{2.2}$$

Analogous equations were derived earlier in [3] directly from the initial system of equations (1.14)-(1.15), but the dispersion relations were not analyzed.

There follows from (2.1)

$$\beta\,\mathrm{Re}=-\frac{3}{2}\frac{c^*-3}{c^*-1.2}. \tag{2.3}$$

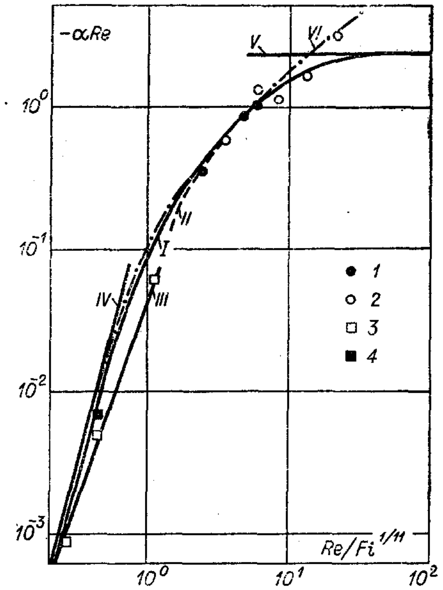

Fig. 4

Eliminating β from (2.2) by using (2.3), we derive the quadratic equation in k^2Re^2 whose solution is

$$(k\,\mathrm{Re})^2 = \frac{\mathrm{Re}^3}{6\,\mathrm{We}}(c^* - c_1^*)(c^* - c_2^*)\left[1 \pm \sqrt{1 + \frac{27\,\mathrm{We}}{\mathrm{Re}^3}\frac{(c^*-3)(c^*+0.6)}{(c^*-1.2)^2(c^*-c_1^*)^2(c^*-c_2^*)^2}}\right], \tag{2.4}$$

where $c_1^* = 1.69$ and $c_2^* = 0.71$.

Neutral waves exist under the conditions

$$\beta = 0,\ c^* = 3,\ k = \sqrt{\mathrm{Re/We}}. \tag{2.5}$$

Waves with $c^* > 3$ damp out exponentially, and with $c^* < 3$ grow.

An analysis of (2.3) shows that the maximum of the increment in β corresponds to the minimum phase velocity on the dispersion curve (2.4). For an exact determination of the maximum-growth wave characteristics, we turn to (2.2) and (2.3). We rewrite (2.3) as

$$c^* = 1.2 + 1.8/\Phi, \tag{2.6}$$

where $\Phi = 1 + 2\beta\,\mathrm{Re}/3 \geq 1$. We then substitute (2.6) into (2.2), we differentiate the expression obtained with respect to k and taking account of the extremum condition $\partial\Phi/\partial k = 0$, we obtain

$$k\,\mathrm{Re} = 0.2\sqrt{\frac{\mathrm{Re}^3}{\mathrm{We}}}\sqrt{\frac{13.5}{\Phi^2} - 1} = \sqrt[4]{\frac{3}{4}\frac{\mathrm{Re}^3}{\mathrm{We}}(\Phi^2 - 1)}. \tag{2.7}$$

Finally, substituting (2.6), (2.7) into (2.2), we arrive at the expression

$$\frac{\mathrm{Re}^3}{\mathrm{We}} = \frac{\Phi^4(\Phi^2-1)}{(\Phi^2-13.5)^2}\frac{3\cdot 10^3}{6.4}. \tag{2.8}$$

The increment of the maximum-growth wave is

$$\beta = 1.5(\Phi - 1)/\mathrm{Re}. \tag{2.9}$$

In order to go from the time increments β to the space increments (−α), which are indeed measured in experiment, the known Gaster transformation must be used

$$-\alpha = \beta\Big/\frac{\partial\omega}{\partial k} = \beta\Big/\left[c^* + k\frac{\partial c^*}{\partial k}\right] = \beta/c^*. \tag{2.10}$$

It is here taken into account that $\partial c^*/\partial k = 0$ for the maximum-growth waves. As a numerical computation has shown, within the framework of the problem formulated, the spatial and time increments actually are related sufficiently accurately by (2.10).

Therefore, the group of formulas (2.6)-(2.10) describes all the characteristics of waves growing maximally in a film. Since the numbers Re and We are in the formulas in the combinations Re^3/We, it is convenient to convert it so that only one rated parameter Re would be used

$$\mathrm{Re}^3/\mathrm{We} = 3^{2/3}(\mathrm{Re}/(\mathrm{Fi}^{1/11})^{11/3}.$$

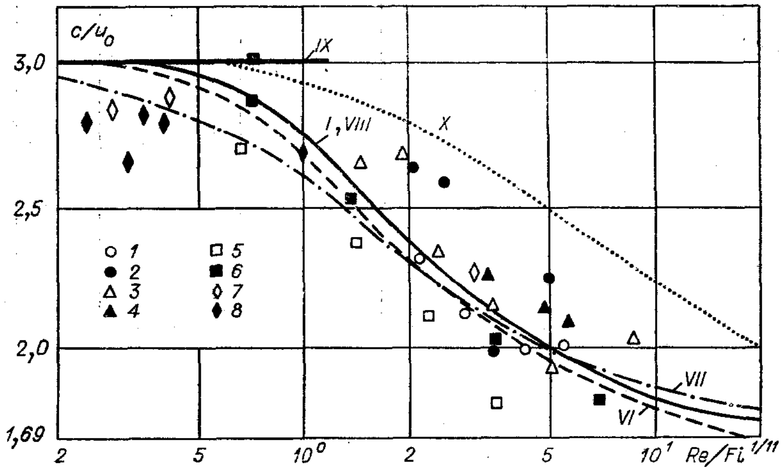

Fig. 5

3. EXPERIMENTAL TECHNIQUE

Experiments to investigate waves in their domain of origin were performed on the apparatus of [12, 13]. Fluid went from a constant level tank to the outer surface of the working section where it flowed in the form of a film, through a system of rotameters. An organic glass tube of 60-mm outer diameter and 1-m length was used as working section. The fluid was delivered to the working section through an annular slot of a distributor whose length was 70 mm and its width was 0.5-1 mm.

The main difficulty in preparing the experiment was in organizing the two-dimensional flow of the fluid wave film. In order to obtain uniform trickling, the working sections were mounted strictly vertically, and the magnitude of the annular gap and the coaxiality of the distributor with the working section was regulated precisely until the waves originating become two-dimensional (annular). The possibility of such regulation was achieved because of the slight play between the fitted surfaces of the working section and the distributor.

The fluid film is extremely responsive to external perturbations, in particular to vibrations from the working pump. Hence, the tests were performed only with the pump disconnected, and the fluid was pumped in the constant head tank periodically in an automatic mode.

Water—glycerine and water—alcohol solutions which possess the advantage that they are very much less subject to the influence of pollutants being adsorbed on the film surface as compared to pure water were used as working fluids.

The instantaneous and mean thickness of the film, and the amplitude, velocity, and length of the waves were measured during the experiments. The film thickness was measured by a shadow method which consisted of photographing the shadow cast by the fluid film upon illumination of the working section [12]. The wave phase velocity was determined by the phase shift between two simultaneous inscriptions of the instantaneous film thickness, which corresponded to two different points along the tube. The accuracy of measuring the absolute thickness was estimated as 2-5%, and the phase velocity as 5-9%.

4. TWO-DIMENSIONAL WAVES IN THE DOMAIN OF THEIR ORIGIN

The fluid film flow pattern along the vertical surface for the numbers Re = 5-50 has the following form. The fluid film is smooth directly at the exit slot. Then at a certain distance from the edge of the slot, infintesimal two-dimensional periodic perturbations originate because of the natural instability of the smooth laminar flow, and grow in amplitude quite rapidly. Upon reaching sufficiently high amplitudes the nonlinearity starts to influence, and the waves emerge in the nonlinear stationary mode. Two-dimensional waves are themselves unstable and rapidly develop into three-dimensional horseshoe-shaped perturbations which are naturally nonstationary.

Results on the evolution of two-dimensional waves in the domain of their origin are presented in Figs. 2 and 3. Oscillograms of the film thickness were taken at different distances from the exit slot edges by advancing the optical system along the working section.

It follows from Fig. 2 that the originating waves are sinusoids and, as has been shown earlier, their amplitude grows exponentially with distance. And only upon reaching sufficiently high amplitudes (at the distance $x/h_0 \sim 200$, Fig. 2) is the wave shape distorted and the growth rate retarded abruptly. Data on the velocity and wavelength of growing waves are represented for convenience in the form of dependences on the amplitude (Fig. 3), from which the linearity of the waves in the domain of their origin follows directly. The amplitude a in Fig. 3 is defined as the difference between the maximum and minimum values of the thickness.

It must be noted that the originating waves are not generally strictly regular, and hence, statistical processing of the signal is required to obtain the average wave characteristics. However, comparatively regular two-dimensional waves can be observed in the wave formation domain for a well-organized fluid delivery to the working section and suitable fluid properties (glycerine and alcohol solutions) and mass flow rates. Namely such wave modes were indeed processed in this experiment where possible.

According to linear instability theories, the waves observed in practice near the wave formation line should correspond to maximum-growth waves, as is partially confirmed in [18, 19] for the velocity and wavelength. Data on the increment, velocity, and length of the growing waves are presented in Figs. 4-6, and are compared with theoretical dependences for maximum growth waves and the experimental data of other authors. In constructing the graphs, coordinates were selected in which the theoretical dependences obtained in this paper are universal curves.

In the range of numbers Re > 10 our experimental data 1 in Fig. 4 for the water–glycerine solution ($\nu = 2.34 \cdot 10^{-6}$ m²/sec, $\sigma/\rho = 60.2 \cdot 10^{-6}$ m³/sec², $Fi^{1/11} = 6.4$) agree with the data 2 [20]. For the number Re ≤ 1 the experimental data 3, 4 from [4] are superposed for an oil film.

For values of the complex $Re/Fi^{1/11} < 0.5$ the experimental points are described well by different theories: I is a computation using (2.6)-(2.10), III is from [1], IV is the long-wave approximation in [1], and VI is from [5]. In the moderate Re number range, test data are generalized for $Re/Fi^{1/11} > 2$ by the theoretical dependences I, VI and partially by a computation II in [2]. For $Re/Fi^{1/11} > 50$ the dependence I agrees with the computation V in [17].

Our data on the velocity and wave number of growing waves are presented in Figs. 5 and 6 for glycerine and alcohol solutions: 1) $\nu = 2.12 \cdot 10^{-6}$ m/sec, $\sigma/\rho = 65.3 \cdot 10^{-6}$ m³/sec²; 2) $2.12 \cdot 10^{-6}$ and $28.5 \cdot 10^{-6}$; 3) $3.72 \cdot 10^{-6}$ and $61 \cdot 10^{-6}$; 4) $2.34 \cdot 10^{-6}$ and $60.2 \cdot 10^{-6}$. The numbers 5, 6 denote the data of [18, 19] for growing waves on a vertical water film, respectively, while 7, 8 are data [4] for waves growing maximally on oil films for Re ≤ 1. The experimental points in Fig. 5 have a considerable spread, which is due to the difficulty in measuring the wave characteristics of low-amplitude, very shallow waves. Nevertheless, reasonable agreement is observed between experiment and linear theories of maximal growth waves. Besides the dependences I constructed by formulas (2.6)-(2.8), the following theoretical dependences are presented in Figs. 5 and 6 for maximum-growth waves, obtained numberically: II from [2] for water; VI from [5] for water; VII, VIII from [4] for oil ($Fi^{1/11} = 1.72$) and water ($Fi^{1/11} = 9.2$), respectively. Curves I and VIII are practically coincident, although calculated on the basis of different equations. Here the neutral curves IX from (2.5) and X from [21] are presented.

Therefore, it follows from Figs. 2-6 that the behavior of growing waves in the wave formation domain is described at the initial stage of their evolution by linear theories of maximum-growth waves.

The dependences I in Figs. 4-6 generalize the experimental points sufficiently well despite the simplicity of the equations used to derive them, and agree with other theories in a broad range of variation of the complex $Re/Fi^{1/11}$, which is one of the proofs of the universality of the two-wave equation (1.26).

Assuming the results of the numerical computation of the Orr–Sommerfeld equation in [5] to be valid in the whole range of Re numbers presented, a more exact derivation of the boundaries of applicability of the boundary-layer approximation for a wave fluid film can be made from a comparison of the dependences I and VI, say, in Fig. 4. As is seen from Fig. 4, the best agreement between the dependences I and VI is observed in the range of values of the complex $Re/Fi^{1/11} = 1$-10. However, even for $Re/Fi^{1/11} < 1$ the agreement between the

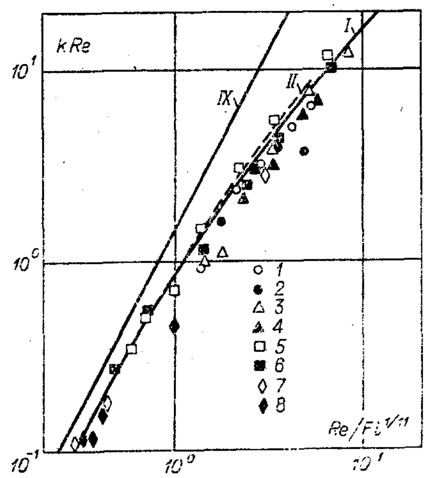

Fig. 6

theories can be considered satisfactory since the dependences differ only by a numerical factor, but have identical asymptotics. A more substantial discrepancy is observed for $Re/Fi^{1/11} > 10$ since the theoretical dependences have different asymptotics in the number Re.

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